Estimating Primaries by Sparse Inversion, a Generalized Approach

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M.Sc. Thesis

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Abstract

For an accurate interpretation of seismic data, multiple-free data is of great value. Removing surface multiples and interbed multiples proves to be challenging in many cases. The nowadays widely used method of Surface-Related Multiple Elimination (SRME) has lately been extended with an inversion-based feedback-loop, resulting in the method of Estimation of Primaries by Sparse Inversion (EPSI). The new method was shown to be more accurate than the former method in many situations. It especially proved valuable in shallow water marine data, because of the strong water-bottom multiples overlapping with subsurface primaries, which could give erroneous results in the SRME case. Furthermore, because the EPSI method estimates primaries from multiple information, near offset reconstruction is included in the multiple removal step.

In this thesis, the EPSI model is generalized to remove both surface and interbed multiples. Removing surface multiples is a special case in this new parameterization. The new method performs a double iteration to explain all data in terms of a primary impulse response and a source signature, removing both surface and interbed multiples.

Like the original EPSI method, the newly proposed scheme requires some knowledge about the subsurface; the data should be divided into (macro) layers and appropriate time windows must be available.

The method is tested on two 2D synthetic data sets and one 2D field data set, with satisfactory results.
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Chapter 1

Introduction

1.1 Seismics

The study of seismology tries to get a good understanding of what the earth looks like under the surface. It is of great importance in, for example, the search for fossil fuels (exploration seismology) and the investigation of the earth with respect to predicting earthquakes (global seismology). In this thesis, we will consider the first case; the exploration seismic method. In an attempt to obtain information about the subsurface, conventionally, three stages are discerned, namely data acquisition, processing and interpretation. The processing step is about manipulating the gathered data such that we obtain specific accurate information about the subsurface. The output is most commonly in the form of images.

The steps that are taken during the data processing stage are based on the understanding of the physical processes that are involved in seismic experiments. We will be focusing on a specific step in data processing, namely the multiple removal step.

1.2 Multiples

In the field of active seismology, signals are sent into the earth, and their reflections are measured by either geophones on land, or hydrophones on water. These signals travel as waves through the subsurface, and get diffracted and reflected as the properties of the different layers in the earth vary. Simply said, one is looking at echoes of the source signal that were created by the subsurface. Seismologists try to translate the data they have measured at the surface to the properties of the earth as a function of position in the earth.

Figure 1.1 shows a typical seismic measurement with some paths depicted that the source signal has traveled before it was measured. The wave paths that have been drawn in blue show signals that have reflected once on a subsurface boundary and traveled back directly to the surface, where they were picked up. We call these signals primaries. The time between the moment of stimulation and the moment of measurement tells something about the depth of the reflecting layer boundary. In conventional data processing, we would like to have an as clear image of the primaries as possible. This information is interpreted to form an image of the subsurface layers and their properties.

A downside of looking at echoes is that reflected waves can reflect a second, or a third time, or even more. An upcoming wave that reflects down and up again is called a multiple. Multiples from
Chapter 1. Introduction

Figure 1.1: Subsurface model with various ray paths. We can distinguish **primaries**, **surface multiples** and **interbed multiples**.

shallow parts of the earth overlap with primaries from deeper parts, sometimes making the latter indistinguishable. Therefore, multiples are of specific concern to a geophysicist specialized in data processing. They should be removed from the original measurement before the data can accurately be interpreted.

This thesis distinguishes two types of multiples: multiples that have reflected downward once or more times at the earth’s surface are called **surface multiples** (marked green in Figure 1.1). These types of multiples are observed especially in measurements at sea; the water surface is a strong reflector, with a reflection coefficient close to $-1$. All other multiples are called **interbed multiples** (marked red in Figure 1.1).

Several multiple removal techniques have been developed. In the method of SRME (Surface Related Multiple Elimination), introduced by Verschuur et al. (1992), and its interbed-removing counterpart IME (Internal Multiple Elimination) as described by Berkhout and Verschuur (2003), these multiples are predicted using the physical explanation of their occurrence. The predicted multiples are then subtracted from the measurement to obtain a multiple-free image of the subsurface. SRME and IME have proven to be valuable and have established themselves as two of the standard multiple removal tools in today’s seismic data processing sequences.

1.3 Feed-back through Inversion

However, SRME does not always produce perfect results. Figure 1.2 shows a simple example illustrating a flaw of SRME. The adaptive subtraction step combined with the L2 norm (i.e., minimum energy, see also Verschuur and Berkhou, 1997) can give erroneous results when multiples overlap with primaries. Moreover, the algorithm does not check whether the multiple-free solution is consistent with the information that was gathered by predicting the multiples.

Instead of considering multiples simply as noise (which one would like to remove from a measurement), van Groenestijn and Verschuur (2009) proposed to look at multiples as a source of information. Multiples contain information about the subsurface as much as the primaries do. In fact, multiples have traveled at least twice through the same part of the subsurface, which would make them twice as sensitive to small details in the propagation and reflection effects of the earth. Instead of performing multiple prediction and subtraction, one might try to create and estimate a parameterization of the subsurface which explains the primaries and the multiples observed in the total measurement. In the proposed method, primaries are estimated with use of the information stored in the surface multiples.
1.3. Feed-back through Inversion

Figure 1.2: Synthetic dataset with SRME results (from Verschuur, 2006). (a) Shot record. (b) True surface multiples. (c) True primaries (including interbed multiples, which are treated as primaries). (d) Estimated surface multiples with SRME. (e) Resulting primaries after multiple subtraction. Because of the overlap of strong multiples with some of the primaries, the minimum energy norm is a noticeable false assumption. In this case some multiples (for instance, compare the multiple in (b) and (d) at 1.0s) are over-estimated, resulting in leakage at the indicated places (compare (c) and (e)).
Through inversion, the result is constantly compared to the original measurement, and inconsistencies are fed-back into the algorithm. This technique is called EPSI (Estimating Primaries by Sparse Inversion).

1.4 Thesis Outline

This thesis will treat a generalized version of the EPSI method. Van Groenestijn and Verschuur (2009) only considered the case of surface multiples. The extended EPSI method can now address surface multiples as well as interbed multiples. For completeness, Chapter 2 will start with basic physics, gradually moving towards the generalized EPSI method, which is proposed in Chapter 3. Simple examples and results will be presented in the form of figures to clarify the theory.

In Chapter 4, results on two different data sets, one of them synthetic, the other containing field data, are presented to show the advancements made in this research. The last chapters provide conclusions and recommendations, giving a short reflection on this research and proposals on possible future research.
Chapter 2

Theory of Multiple Prediction

Removing the multiples from a seismic measurement data set enables us to have a better view on the valuable information stored in the primaries. Multiple-free data is ‘cleaner’ and therefore more easily interpreted by a seismologist. Two categories of multiple removal techniques can be considered (see also Verschuur, 2006):

1. Methods based on a difference in spatial behavior of primaries and multiples;
2. Methods based on periodicity and predictability.

Methods from the first category differentiate primaries from multiples by the fact that they traveled a different path through the earth. The arrival times and wavefront shapes of these types of events can thereby be distinguished. By applying a filtering technique that separates the multiples from the primaries, the multiples can be muted out. In most cases, the data is transformed to a specific domain, for example the F-K domain (Ryu, 1982) or the parabolic Radon domain (Hampson, 1986), where the multiples and primaries map to a different area. By suppressing the ‘multiple area’ and performing an inverse transformation, multiple-free data can be obtained.

However, the assumptions made in these methods often do not hold. For example, multiples do not exhibit a lower move-out velocity than primaries in all cases, and not all events can be described by a parabola or hyperbola in the CMP-offset domain (Verschuur, 2006). Therefore, in many cases these methods are not adequate.

Methods from the second category exploit the fact that multiples are simply a repetition of primaries, and higher-order multiples are a repetition of lower-order multiples. The repetitions are predicted and suppressed to remove the multiples from the data. The methods of SRME (Verschuur et al., 1992) and EPSI (van Groenestijn and Verschuur, 2009) fall into this category. These methods both build on the technique of wave field extrapolation. They are data-driven approaches in terms of multi-record convolution (Berkhout, 1982).

2.1 The Kirchhoff-Helmholtz Integral

The Dutch physicist Christian Huygens (1629-1695) is known in the field of acoustics for his Huygens’ principle. This is actually the underlying principle for wave field extrapolation. It states that, when a wave front is known on a specific time instant, the wave front on the next time instant can be constructed from the envelope of wave fronts of secondary sources that were put at the original wave
Figure 2.1: Schematic representation of Huygens’ principle. Each point of a wave front (solid red line) can be seen as the center of a new secondary source (blue). Consequently, the advancing wave front (dashed red line) as a whole may be regarded as the sum of all the secondary wave fronts from the wave front in an earlier stage. Note that in this figure the wave front could either be traveling outwards or inwards.

Figure 2.2: Given a closed volume $V$, containing no sources, we can synthesize the pressure field in point $A$ inside the volume knowing the wave field on the surface of $V$, noted as $S$.

front, illustrated in Figure 2.1. He was not specific about the direction of propagation, nor did he describe the amplitudes of the new wave front, but his findings hold in an intuitive way.

Gustav Kirchhoff (1824-1887) performed a more thorough investigation on predicting a wave field from an existing measurement. He showed that, when one knows the wave field outside a closed volume, the wave field inside can be predicted (see Figure 2.2). In fact, when addressing a closed volume $V$ having a surface $S$, knowing the pressure field on $S$ and its derivative with respect to the normal vector $\vec{n}$ is sufficient to predict the pressure in every point $A$ in $V$. This can be computed by what we know as the Kirchhoff-Helmhotz integral (Berkhout, 1982):

$$P_A = \frac{-1}{4\pi} \oint_S \left[ P \frac{\partial G}{\partial n} - G \frac{\partial P}{\partial n} \right] dS,$$

(2.1)

where $P_A$ is the pressure field in point $A$ in the frequency domain, and $\vec{n}$ is the normal vector at the surface $S$ that points outward. $G$ is called a Green’s function, and describes the response of a point source in $A$ measured at the surface $S$. Both $G$ and the pressure $P$ on $S$ are functions of position, the medium properties and temporal frequency.
2.2 The Rayleigh II Integral

Using the equation of motion (see Berkhout, 1982), we know that the factor \( \frac{\partial p}{\partial n} \) (\( p \) being the pressure field in the time domain) is proportional to the particle acceleration along the vector \( \vec{n} \):

\[
\frac{\partial p}{\partial n} = -\rho_0 \frac{\partial v_n}{\partial t}, \tag{2.2a}
\]
or, in the frequency domain:

\[
\frac{\partial P}{\partial n} = -j\omega \rho_0 V_n, \tag{2.2b}
\]

where \( \rho_0 \) denotes the static density of the medium. Equations 2.2 are linearized approximations. It was shown by Berkhout (1982) that in most practical situations, these linearized versions may be used. Substituting Equation 2.2b in Equation 2.1, we can see that we need to know the pressure field \( P \) and the particle velocity \( V \) along \( \vec{n} \) on \( S \) to predict the pressure field in the enclosed volume:

\[
P_A = \frac{1}{4\pi} \oint_S \left[ P \frac{\partial G}{\partial n} + (j\omega \rho_0 V_n)G \right] dS, \tag{2.3}
\]

where we have chosen an inward pointing \( \vec{n} \) to eliminate the minus sign of Equation 2.1.

The result we have now is unfortunately not very useful for the seismic situation. We hardly ever know both the pressure field and the particle velocities on a closed surface around our volume of interest, therefore, we will have to simplify the situation.

In a typical seismic measurement the wave field is measured at a certain horizontal (flat) surface. Let us consider the situation illustrated in Figure 2.3. We have split the surface \( S \) in a plane \( S_1 \) and a hemi-sphere \( S_2 \). When we now assume that the pressure field in the lower half space is generated by causal sources in the upper half space, we can always choose \( R \) such that the contribution from \( S_2 \) has not yet reached point \( A \). So if we need to know \( P_A \) for \( 0 < t < T_{\text{max}} \), Equation 2.3 simplifies to

\[
P_A = \frac{1}{4\pi} \int_{S_1} \left[ P \frac{\partial G}{\partial n} + (j\omega \rho_0 V_n)G \right] dS_1, \text{ for } R > cT_{\text{max}}. \tag{2.4}
\]

Assuming that the medium below \( S_1 \) is homogeneous, having a propagation velocity \( c \), we know from Berkhout (1982) that \( P \) should satisfy the homogeneous wave equation

\[
\nabla^2 P + k^2 P = 0, \tag{2.5}
\]
for all points inside \( V \), and that \( G \) must be a solution of an extended version of the wave equation:

\[
\nabla^2 G + k^2 G = -4\pi \delta(\vec{r} - \vec{r}_A),
\]

which describes a compressional wave field with a singularity in \( A \) given by a monopole source. We have introduced the wave number \( k = \omega/c \), and used a spatial delta pulse \( \delta(\cdot) \). A possible solution for Equation 2.6 can be constructed from the pressure for a compressional wave field generated by a monopole at \( A \), plus a second term \( H \):

\[
G = \frac{e^{-jkr}}{r} + H,
\]

with \( \vec{r} = (x_A - x, y_A - y, z_A - z) \), and its length denoted by \( |\vec{r}| = r \). We can choose any function for \( H \), as long as it satisfies the wave equation

\[
\nabla^2 H + k^2 H = 0,
\]

inside \( V \) and on \( S_1 \), i.e., it should have no singularities inside \( V \) nor on \( S_1 \).

Now, it would be interesting to find a function \( G \) such that \( G = 0 \) for every point on \( S_1 \). Obviously, this would eliminate the second term inside the integral in Equation 2.4, simplifying the problem even more. Let us use the following Green’s function:

\[
G = e^{-jkr} - \frac{e^{-jkr'}}{r'},
\]

with \( \vec{r}' = (x_A' - x, y_A' - y, z_A' - z) = (x_A - x, y_A - y, -z_A' - z) \). Note that \( r' = r \) on \( S_1 \), i.e., the lengths of the vectors are equal, see Figure 2.3. It can be easily seen that the choice of \( H = -\frac{e^{-jkr'}}{r'} \) satisfies both Equations 2.6 and 2.8.

Substituting Equation 2.9 in Equation 2.4 produces

\[
\begin{align*}
P_A &= \frac{1}{4\pi} \int_{S_1} P \frac{\partial}{\partial n} \left( \frac{e^{-jkr}}{r} - \frac{e^{-jkr'}}{r'} \right) dS_1 \\
&= -\frac{1}{4\pi} \int_{S_1} P \frac{1 + jkr}{r^2} \left( \frac{\partial r}{\partial n} - \frac{\partial r'}{\partial n} \right) e^{-jkr} dS_1 \\
&= \frac{1}{2\pi} \int_{S_1} P \frac{1 + jkr}{r^2} \cos \phi \ e^{-jkr} dS_1,
\end{align*}
\]

where \( \cos \phi = z_A/r \). Equation 2.10 is known as the Rayleigh integral II (see, a.o., Berkhout, 1982).

A similar derivation exists when we like to build up the wave field in \( A \) by using the particle velocity \( V_n \) at the surface \( S_1 \), resulting in the Rayleigh integral I (see Berkhout, 1982). We will not go into details on this.

As seismic recordings are often 2-dimensional instead of 3-dimensional, let us assume that \( P \) is independent of \( y \). It is shown by Berkhout (1982) that for this case Equation 2.10 changes to the 2D Rayleigh integral II:

\[
P_A = \sqrt{\frac{j}{2\pi}} \int_{L_x} P \cos \phi \ e^{-jkr} dx,
\]

where we have taken a far-field approximation, using \( kr \gg 1 \), and \( P \) is now independent of \( y \). We can immediately identify the expression for the far-field approximation of a 2-dimensional (i.e., cylindrical) dipole response in a homogeneous medium in this equation (see Berkhout, 1982); being \( \sqrt{2\pi jk} \cos \phi \ e^{-jkr'/r'} \). This expression includes a factor \( \sqrt{j} \), which represents a 45 degree phase shift, shown by

\[
\sqrt{j} = e^{j\frac{\pi}{4}}.
\]
2.3 Inverse Wave Field Extrapolation

What we have shown so far, is that we can construct the pressure field in a point beneath a flat surface, knowing only the pressure field on that surface. We can build up this field by synthesizing the wave field of a dipole distribution on the surface. In more practical terms, when we have measured a wave field at a surface $S_1$, we can reconstruct the wave field in every point $A$ below $S_1$ by applying a time shift to the wave field (the exponential term in Equations 2.10 and 2.11) and an amplitude factor. The integral sums all time shifted and scaled measurements to provide the extrapolated wave field in $A$. This result holds for a homogeneous subsurface, as stated before. The sources must be causal, and the time window of proper prediction is dependent on the shortest distance from point $A$ to the edge of the plane.

In Figures 2.4 and 2.5, some basic examples are given to illustrate the process of wave field extrapolation. In Figure 2.4, a single trace from a source to a point $A$ below a certain depth level is constructed, using the wave field at that depth level, and the Green’s function from the point $A$ to the level. Note that the result (Figure 2.4d) is precisely what one would expect, except for a small artifact at around 0.65s. This can be explained by the fact that the boundary condition stated in Equation 2.4 is not fulfilled. By widening the measurement area, i.e., by using larger offsets, these artifacts could have been removed from the result (actually, they would shift outside the time-window of 1s).

Figure 2.5 shows the result for wave field extrapolation to a dipping level. The artifact we saw in Figure 2.4d shows up in Figure 2.5b as two extra ‘tails’ from the edges of the wave field.

2.3 Inverse Wave Field Extrapolation

It is also possible to reconstruct the wave field at a point where the field has already passed. As a practical example, when we measure the reflected wave field from an inhomogeneity in the subsurface at the surface, we would like to back-propogate it to the reflection interface. The same equations can be used as for the case of forward wave field extrapolation, with only a minor adjustment. We must use a time-reversed version of the Green’s function $G$. In the frequency domain, this means we should take the complex conjugate of $G$, i.e., we should use $G^*$ in Equations 2.1 and 2.4. This will result in

$$P_A = \sqrt{-\frac{j}{2\pi}} \int_{L_x} P \cos \phi \frac{e^{ikr}}{\sqrt{r}} dx,$$

which includes a factor $\sqrt{-j}$, i.e., a $-45$ degree phase shift.

Figures 2.6 and 2.7 show us the process of inverse wave field extrapolation. In Figure 2.6, a single trace from a source to a point above a certain depth level is constructed, using again the wave field at that level, and the Green’s function from the point to the level. In the result in Figure 2.6d, an artifact shows up at about the same time as the main wavelet (at about 2.5s), distorting the latter.

In Figure 2.7b, we see that with inverse wave field extrapolation, the ‘extra tails’ we noted earlier show up just above the top of the wave field. However, a bigger error is made by the fact that we use data from a relatively small aperture to reconstruct a wave field with a larger aperture. This will result in bad illumination effects at large offsets. In Figure 2.7b, we can see that at large offsets, both the arrival times and the amplitude of the wave do not resemble the exact solution in Figure 2.7c. Again, we can conclude that collecting data from larger offsets would increase our accuracy for wave field extrapolation.
Figure 2.4: Example of forward wave field extrapolation from a depth level to a point. (a) Measured wave field at a depth level of 500m. (b) Green’s function from point A to the depth-level. (c) Trace by trace convolution result of the wave field with the Green’s function. (d) All traces in (c) summed together form the stationary result of the wave field as if it had traveled from the source position to point A. (After Verschuur, 2006)
2.3. Inverse Wave Field Extrapolation

Figure 2.5: Example of forward wave field extrapolation from a depth level to another (dipping) level. This result can be obtained by repeating the process from Figure 2.4, letting point A move along the second level. (a) Measured wave field at a depth level of 500m. (b) Wave field extrapolation result from the depth level of 500m to the dipping level. The trace from Figure 2.4d shows at the lateral position of 200m. (c) Direct wave field from the source to the dipping level. Note the resemblance between (b) and (c), except for some small artifacts that appear as extra ‘tails’ from the edges of the wave field. (After Verschuur, 2006)
Figure 2.6: Example of inverse wave field extrapolation from a depth level to a point. (a) Measured wave field at a depth level of 500m. (b) Green’s function from point A to the depth-level. (c) Trace by trace convolution result of the wave field with the time-reversed Green’s function. (d) All traces in (c) summed together form the stationary result of the wave field as if it had traveled from the source position to point A. Note that the shape of the wavelet has been distorted due to boundary artifacts. (After Verschuur, 2006)
2.3. Inverse Wave Field Extrapolation

This result can be obtained by repeating the process from Figure 2.6, letting point A move along the second level. (a) Measured wave field at a depth level of 500m. (b) Inverse wave field extrapolation result from the depth level of 500m to the dipping level. (c) Direct wave field from the source to the dipping level. The resemblance between (b) and (c) is in this case less convincing, as compared to Figure 2.5 because of poor illumination. (After Verschuur, 2006)
2.4 Model-driven Prediction of Surface Multiples

When we have substantial knowledge about the reflection interfaces in the subsurface, we can use wave field extrapolation to predict surface multiples. By simply subtracting these multiples from the original data, it is possible to remove them from the measurement. This procedure forms the basis for the multiple removal techniques that will be covered in this thesis.

The prediction of surface multiples is carried out in two steps. We must first forward extrapolate the measured shots to the first reflection boundary. This is done by convolving the measured shots with Green’s functions that describe the propagation from the interface to the shot positions, as shown before in Figure 2.5. The results are multiplied by a factor of $-1$ to represent reflection against the underside of the surface. Another forward extrapolation back to the surface creates the multiples we are looking for. The Green’s functions for this second step resemble the ones in the first step, though the sources are now placed at the surface and the receivers at the subsurface interface. The full procedure is illustrated in Figure 2.8.

The primaries in the original data are now mapped to first-order surface multiples, first-order surface multiples are mapped to second-order surface multiples, etcetera. Using an adaptive subtraction method, we can match the amplitudes of the predicted multiples to the amplitudes of the measured multiples in the original data, and produce our surface multiple-free data.

Because we have only extrapolated to the first interface, we have not yet removed surface multiples generated by the second reflector. This can be done in a similar manner, though we should now use a Green’s function that incorporates the inhomogeneity of the subsurface, which is usually not available at the required accuracy.

2.5 Using the Primary Impulse Response

The two-step extrapolation of a seismic data set as shown in Section 2.4 adds another round trip of the signal to the first reflection boundary in the subsurface and back to the surface again. The same result can be obtained when the data is convolved with the primary impulse response of the first reflector. Unlike the double extrapolation step, this procedure also accounts for the angle- and position-dependent reflection coefficient of the boundary, which are contained in the impulse response.

Let us define some parameters for a mathematical description of our seismic data. Using the detail-hiding operator notation for 2D data (Berkhout, 1982), we can split a measurement $P$ into the surface multiples $M_0$ and all other events $P_0$ (the latter containing the primaries and all interbed multiples):

$$P = P_0 + M_0. \quad (2.14)$$

The symbols in this equation represent matrices, a short explanation of this notation is given in A.

We will distinguish interbed multiples based on the shallowest boundary that they reflected back down into the earth on:

$$P_{n-1} = P_n + M_n, \quad (2.15)$$

where $M_n$ represents all multiples that have the $n$-th reflector as their shallowest downward reflector. The water surface can be interpreted as the 0-th reflector for consistency with Equation 2.14.

Furthermore, let us define the impulse response $X$ and source signature $S$ as:

$$P_n = X_n S. \quad (2.16)$$
Figure 2.8: Illustration of forward propagation to the first depth level in a simple three reflector model. The values inside the subsurface figure represent pressure-wave velocity in m/s. This procedure consists of two forward wave field extrapolation steps as illustrated in Figure 2.5b. (a) Shot record of the total up-going data $P$ for the three reflector model shown above. (b) Forward extrapolation of the data to the first depth level. This step uses Green’s functions from the depth level to the surface (i.e., the other way the blue arrows are pointing). (c) Forward extrapolation of the data from (b) to the surface produces the set of surface multiples we were looking for. This step uses Green’s functions from the surface to the first depth level. (d) Data from (a) minus the surface multiples from (c). All data has been generated directly from the model, so no irregularities are showing. The double extrapolation procedure is the same as a multi-record convolution with the primary impulse response of the first reflector $\hat{X}^{(1)}$ (visible in (c) by the connected blue arrows).
Chapter 2. Theory of Multiple Field Prediction

Figure 2.9: Multiple prediction. a) Surface multiples can be predicted from two wave field components. The multiple $\hat{\mathbf{M}}^{(nm)}$ is constructed by $\hat{\mathbf{X}}^{(n)} R_0 \hat{\mathbf{P}}^{(m)}$. b) An interbed multiple can be constructed with three wave field components. The multiple $\hat{\mathbf{M}}^{(nm,l)}$ can be considered as the combination of the two primaries $\hat{\mathbf{X}}^{(n)}$ and $\hat{\mathbf{P}}^{(l)}$ (with raypaths $x_s x'_r$ and $x'_s x_r$, respectively), minus a third primary $\hat{\mathbf{X}}^{(m)}$ (raypath $x'_s x'_r$). The latter is time-reversed, and compensates for the effect of the raypath $x'_s x'_r$ from the combination of $x_s x'_r$ and $x'_s x_r$.

$S$ represents the temporal Fourier transform of the source signature $s(t)$, which has been put into a matrix. This matrix can be a band matrix (having only non-zero values up to a certain distance from its diagonal), a diagonal matrix, or a constant-valued diagonal matrix ($SI$), depending on the assumptions imposed on the source properties. See Appendix B for a more extensive treatment.

The single multiple event that was formed by the primary from the $m$-th interface that reflected back into the earth at the surface and back up again on the $n$-th boundary, $\hat{\mathbf{M}}^{(nm)}$, can be predicted by a matrix multiplication (thus, a multi-record temporal convolution) of the primary impulse response of the $n$-th reflector $\hat{\mathbf{X}}^{(n)}$ and the primary response of the $m$-th reflector $\hat{\mathbf{P}}^{(m)}$: \begin{equation}
\hat{\mathbf{M}}^{(nm)} = \hat{\mathbf{X}}^{(n)} R_0 \hat{\mathbf{P}}^{(m)}. \tag{2.17}
\end{equation}

This operation is illustrated in Figure 2.9a. The factor $R_0$ represents the reflection operator of the surface from below, which we assumed to be $-1$ (or $-I$) in the previous section. The indexed double hat above the quantities indicate a double mute; above and below the indicated event by the index, showing only that event, see for example the indicated events in Figure 2.8a. If we extend Equation 2.17 to construct all surface multiples, we must include all primary and interbed multiple events in the impulse response, and use the full up-going wave field $\mathbf{P}$:\begin{equation}
\mathbf{M}_0 = \mathbf{X}_0 R_0 \mathbf{P}. \tag{2.18}
\end{equation}

An illustration of this multiple prediction operation can be found in Appendix A. An important observation is, that, due to the dipole characteristics of the 2D Rayleigh Integral II (Equation 2.11), our primary impulse response matrix $\mathbf{X}_0$ also contains dipole source impulse responses.
2.6 Predicting Interbed Multiples

Surface multiples can be predicted using two primary reflections and a reflection operator. From Jacubowicz (1998) we know that by considering three primary reflections, internal multiples can be predicted, as illustrated in Figure 2.9b. We must combine the seismic data with a time-reversed version of the primary associated to the multiple generating interface, and again combine it with the data. In matrix notation:

$$\hat{\mathbf{M}}^{(nm)} = -\hat{\mathbf{X}}^{(n)} \hat{\mathbf{X}}^{(m)} H \hat{\mathbf{P}}^{(l)},$$ (2.19)

where $\hat{\mathbf{M}}^{(nm)}$ represents the interbed multiple that has reflected on interfaces $l$, $m$ and $n$, $\hat{\mathbf{X}}^{(m)}$ and $\hat{\mathbf{X}}^{(n)}$ are the primary impulse responses from the interfaces $n$ and $m$, respectively, $\hat{\mathbf{P}}^{(l)}$ is the primary response from interface $l$, and $(\cdot)^H$ represents the conjugate transpose (or Hermitian transpose). The multiple is decomposed into three components, and can be considered as the combination of two primaries (depicted blue in Figure 2.9b) minus a third primary (depicted light blue). It was noted by Jacubowicz (1998) that the result of Equation 2.19 is exact, with the exception of transmission effects down to and at the interbed multiple generator. We will be neglecting these effects.

It can be shown that the complete set of multiples generated by the $n$-th reflector, $\mathbf{M}_n$, is given by:

$$\mathbf{M}_n = -\hat{\mathbf{X}}^{(n)} \hat{\mathbf{X}}^{(n)} H \hat{\mathbf{P}}^{(n)},$$ (2.20)

where the notation of $\mathbf{P}_n$ and $\mathbf{X}_n$ represents the surface measurement and impulse response, respectively, without the multiples generated by all interfaces from 0 to $n$ (see Equation 2.15). The indexed single hat above the quantities indicate the mute of all primary events from reflectors above and including multiple-generating interface $n$. The terms $\hat{\mathbf{X}}^{(n)}$ and $\hat{\mathbf{P}}^{(n)}$ in Equation 2.20 can contain multiples, hence this expression implicitly includes interbed multiples of other multiples.

When comparing to Equation 2.18, we see that Equation 2.20 incorporates surface multiples as a special case, being the one for $n = 0$, the surface being the ‘zeroth’ reflector with primary impulse response $\hat{\mathbf{X}}^{(0)} = -\mathbf{R}_0$. From a WRW-model point of view (see Berkhout, 1982), this is an obvious statement, because both propagation matrices reduce to the identity matrix when there is no propagation distance.

It must be noted that in the last two sections, the parameter $\mathbf{X}_n$ has sometimes been called an impulse response, and sometimes a primary impulse response. While the latter name is strictly speaking incorrect, we will sometimes use this denotation to emphasize the fact that, when having a fixed $n$, all multiples generated by an interface below the $n$-th interface are treated as if they were a primary. This denotation helps making the distinction between the data we would like to remove (the interbed multiples generated by the $n$-th interface, or, shortly; the multiples), and the data we would like to preserve (the primaries plus all multiples generated by interfaces below the $n$-th interface, or, shortly; the primaries).

A similar approach for multiple prediction as described above was used by Berkhout and Verschuur (2005), where the prediction was performed in the CFP-domain using downward extrapolation. They have employed a data-driven IME variant with extra wavelet corrections.
Figure 2.10: Illustration of Equation 2.20 for n = 1, constructing all interbed multiples generated by the first interface. All figures are shot records. The raypaths above correspond to the highlighted events below. For the impulse responses in (a) and (b), we have included the source signature for better display. (a) Primaries \( \hat{X}(1) \). Note that this term does not contain interbed multiples from the first reflector. The dashed red line represents the muting window. (b) Primary \( \hat{X}(1) \) from the interface multiple generating interface. The muting windows isolate the single primary event. (c) Primaries \( \hat{P}(1) \). Note that this term does not contain interbed multiples from the first reflector. These are mapped to interbed multiples that reflected down twice. (d) The constructed multiples. These are all multiples that reflected downwards on the interface from (b) and upwards on the interface from (a).
Chapter 3

Estimating Primaries by Sparse Inversion

3.1 A More Generalized Approach

In van Groenestijn and Verschuur (2009) the EPSI method is introduced. This method extracts information from the multiples to estimate the primaries. It uses a parameterization that includes a collection of band-limited spikes, representing the primary impulse response. When the estimated parameters are fed into the large-scale inversion system of the EPSI method, they construct the explained primaries and all corresponding multiples, which should add up to the total measured data.

In each step of this iterative process the explained data is sent back into the scheme, so that it can be checked on consistency with the measured data. This way, a feed-back system has been created that ensures that the estimation of the parameters is a valid one.

The minimum energy norm that is used in SRME is replaced by an objective function that will always go to zero.

However, the scheme does not incorporate the removal of interbed multiples. In this chapter, a more generalized version of the EPSI method is proposed.

3.2 Large Scale Inversion

In the EPSI method, the physical model is parameterized in such a way that the parameters explain the total data. This is in contrast with the line of thought behind the SRME model, which tries to separate the measured data into a useful part (the primaries) and noise (the multiples). EPSI estimates the primaries through sparse inversion, thereby avoiding the need to perform an adaptive subtraction of predicted multiples (as is the case in SRME). The forward model is essentially the same as SRME, though it is not based on extracting the primary data $P_0$, but rather on estimating the primary impulse response $X_0$. Combining Equations 2.14, 2.16 and 2.18, the total measured data can be explained by the following parameterization:

$$ P = X_0 S + X_0 R_0 P. \quad (3.1) $$

Equation 3.1 shows that by estimating the parameters $X_0$ and $S$, we are able to reconstruct the surface multiples with the term $X_0 R_0 P$ and the primaries and interbed multiples with $X_0 S$, consequently
explaining the total data.

A generalized approach can be constructed using Equations 2.15 and 2.20:

$$ P_{n-1} = X_n S - \hat{X}_n^{(n)} \hat{P}_n^{(n)} H \hat{P}_{n-1}^{(n)}. $$

(3.2)

This equation enables us to predict interbed multiples using the EPSI method. Equation 3.1 shows the special case of Equation 3.2 for \( n = 0 \), with \( P_{-1} = P \) representing the total data, and the direct primary impulse response of the surface being \( \hat{X}_0^{(n)} = -R_0 \).

Acknowledging the fact that \( X_n = \sum_{m=1}^{n} \hat{X}_n^{(m)} + \hat{X}_n^{(n)} \), the equation that will drive our generalized EPSI method is formed:

$$ P_{n-1} = \left( \sum_{m=1}^{n} \hat{X}_n^{(m)} + \hat{X}_n^{(n)} \right) S - \hat{X}_n^{(n)} \hat{X}_n^{(n)} H \hat{P}_n^{(n)} H \hat{P}_{n-1}^{(n)}, $$

(3.3)

which is reduced to fewer variables by muting the left and right side up to and including event \( n - 1 \):

$$ \hat{P}_{n-1}^{(n-1)} = \left( \hat{X}_n^{(n)} + \hat{X}_n^{(n)} \right) S - \hat{X}_n^{(n)} \hat{X}_n^{(n)} H \hat{P}_n^{(n)} H \hat{P}_{n-1}^{(n)}. $$

(3.4)

As a measure of the error of our estimated parameters, the objective function is introduced:

$$ J = \sum_{\omega} \sum_{j,k} \left| \hat{P}_{n-1}^{(n-1)} - \left( \hat{X}_n^{(n)} + \hat{X}_n^{(n)} \right) S + \hat{X}_n^{(n)} \hat{X}_n^{(n)} H \hat{P}_n^{(n)} H \hat{P}_{n-1}^{(n)} \right|^2_{j,k}, $$

(3.5)

where the summation signs indicate a full summation over all sources, receivers and frequencies. The minimization of the objective function is used as the driving force in the estimation process.

The term inside the absolute square marks is the residual

$$ V = \hat{P}_{n-1}^{(n-1)} - \left( \hat{X}_n^{(n)} + \hat{X}_n^{(n)} \right) S + \hat{X}_n^{(n)} \hat{X}_n^{(n)} H \hat{P}_n^{(n)} H \hat{P}_{n-1}^{(n)}. $$

(3.6)

It represents the unexplained data, i.e., the difference between the total upgoing data and the estimated primaries and corresponding multiples. This quantity goes to zero when all data is explained. Equation 3.5 can also be written as

$$ J = \sum_{\omega} \text{vec}(V)^{H} \text{vec}(V), $$

(3.7)

with \( \text{vec}(\cdot) \) being the standard vectorization transformation of a matrix or vector.

### 3.3 The Sparseness Constraint

A critical constraint used in the EPSI method is the assumption that both \( \hat{X}_n^{(n)} \) and \( \hat{X}_n^{(n)} \) are represented by a collection of band limited spikes in the time domain, denoted by \( \hat{X}_n^{(n)} \) and \( \hat{X}_n^{(n)} \), respectively. Furthermore, these spikes have a certain order in which they should be addressed, which is mainly determined by their amplitude. An iterative scheme is used in which the updates on these parameters are made sparse in the time domain to ensure the correct order. This way, large events are processed before smaller events, iteratively building up the impulse responses.

However, this parameterization differs from the definition of \( X \), which was introduced as a dipole source impulse response in section 2.5. Luckily, the spiky parameters can be ‘converted’ to dipoles.
3.4. The EPSI Procedure

Let us have a closer look at the 2D Rayleigh II integral in Equation 2.11. It was already shown that it contains a dipole impulse response $\sqrt{2\pi jk} \cos(\frac{e^{-jk}}{\sqrt{j\omega}})$. This expression consists of a frequency dependent part $\sqrt{j\omega}$, an amplitude factor, and a time delay.

Selecting a spike from a dipole impulse response $X$ retains the amplitude scaling of the (time delayed) impulse. However, we must correct our spiky impulse response with the frequency dependent factor $\cos(\frac{e^{-jk}}{\sqrt{j\omega}})$ to ensure dipole behavior. By including this factor in every matrix multiplication, a zero phase appearance in the time domain of the spiky parameter $X$ is ensured. Whenever needed, the dipole correction factor $c = \sqrt{j\omega}$ is used to convert this parameter to a dipole impulse response; $X = cX$.

With this new parameterization of the impulse responses and introducing the iteration number $i$, the model from Equation 3.4 looks like:

$$\hat{P}_{n-1}^{(n-1)} = \left(c\hat{X}_{n,i}^{(n)} + c\hat{X}_{n,i}^{(n)}\right)S_i - \omega\hat{X}_{n,i}^{(n)}\hat{X}_{n,i}^{(n)}P_{n-1}^{(n)},$$

where it is easily shown that for the last term, the absolute square of the dipole correction factor equals $c\omega^H = \omega$. The objective function from Equation 3.5 looks like:

$$J_i = \sum_{j,k} \left| \hat{P}_{n-1}^{(n-1)} - c\left(\hat{X}_{n,i}^{(n)} + \hat{X}_{n,i}^{(n)}\right)S_i - \omega\hat{X}_{n,i}^{(n)}\hat{X}_{n,i}^{(n)}P_{n-1}^{(n)}\right|^2,$$

defining the residual from Equation 3.6 as

$$V = \hat{P}_{n-1}^{(n-1)} - c\left(\hat{X}_{n,i}^{(n)} + \hat{X}_{n,i}^{(n)}\right)S_i - \omega\hat{X}_{n,i}^{(n)}\hat{X}_{n,i}^{(n)}P_{n-1}^{(n)}.$$  

The residual is a dependent variable that changes multiples times per iteration. For ease of reading, $V$ is not given an iteration subscript $i$.

### 3.4 The EPSI Procedure

A full EPSI procedure that removes all multiples from a measured data set $P$, starts with $n = 0$ (see Figure 3.1). The three parameters that are to be estimated are $\hat{X}_{n,i}^{(n)}$, $\hat{X}_{n,i}^{(n)}$ and $S$. They are set to zero at the start, and are updated in every iteration. The parameters $\hat{X}_{n,i}^{(n)}$ and $\hat{X}_{n,i}^{(n)}$ are treated as independent parameters, with the restriction that they cannot overlap, imposed by the time-windows. The iterations over $i$ continue until the objective function has come below a certain threshold, or it does not improve any more. Next, $n$ is incremented, moving to the next multiple-generating boundary, and the iterations over $i$ start again. This procedure is repeated to remove all unwanted multiples in a measurement.

As noted before, in Section 2.5, there are various assumptions that can be imposed on the source matrix $S$. The choice depends on the situation and consistency of acquisition. This thesis uses an EPSI implementation with a constant shot source signature assumption, but the theory does not change for the other situations. In appendix B, results are shown of a variable source signature EPSI procedure.

The objective function is minimized in every update step on $\hat{X}_{n,i}^{(n)}$, $\hat{X}_{n,i}^{(n)}$ and $S$ in alternate steps. First, $\hat{X}_{n,i}^{(n)}$ is updated. The update $\Delta\hat{X}_{n,i}^{(n)}$ is created by a steepest-descent step, obtained by taking the derivative of the part of $J$ inside the summation over $\omega$ with respect to the elements of $\hat{X}_{n,i}^{(n)}$:

$$\Delta\hat{X}_{n,i}^{(n)} = cV S_i^H - \omega\hat{P}_{n-1}^{(n)}V H \hat{X}_{n,i}^{(n)}.$$  

(3.11)
Chapter 3. Estimating Primaries by Sparse Inversion

Equation 3.11 shows one of the strong properties of EPSI: the update on the primary impulse response takes information from the unexplained primaries in the residual by the term $c V S^H$, as well as information from the unexplained multiples in the residual by the term $\omega \hat{P}_{n-1}^{(n)} V^H \hat{X}_{n}^{(n)}$. All information is then put into the primary impulse response. This way, EPSI uses information from the measured primaries and multiples simultaneously to estimate the primaries.

A crucial step in the EPSI procedure is the sparseness constraint being imposed on $\Delta \hat{X}_{n}^{(n)}$. After calculation of the update using Equation 3.11, the data is brought to the time domain where the event(s) per trace with the largest amplitude are selected, and a suitable time window is applied (see van Groenestijn and Verschuur, 2009).

The sparse and time-windowed update $\langle \Delta \hat{X}_{n}^{(n)} \rangle$ is now added to the primary impulse response with a frequency-independent scale factor $\alpha$:

$$\hat{X}_{n,i}^{(n+1)} = \hat{X}_{n,i}^{(n)} + \alpha \langle \Delta \hat{X}_{n}^{(n)} \rangle,$$

(3.12)

with $\alpha$ taken as a least-squares scaling factor that minimizes the objective function:

$$\alpha = -\frac{\sum \omega \text{vec}(\Delta V_1)^H \text{vec}(V)}{\sum \omega \text{vec}(\Delta V_1)^H \text{vec}(\Delta V_1)},$$

(3.13)

where

$$\Delta V_1 = -c(\Delta \hat{X}_n^{(n)}) S_i + \omega \hat{X}_{n,i}^{(n)} \langle \Delta \hat{X}_{n}^{(n)} \rangle \hat{P}_{n-1}^{(n)}.$$  

(3.14)

The update on $\hat{X}_n^{(n)}$ is calculated the same way as above, using a steepest-descent step:

$$\Delta \hat{X}_n^{(n)} = V \left( c S_i^H - \omega \hat{P}_{n-1}^{(n)} \beta \hat{X}_{n,i+1}^{(n)} \right).$$

(3.15)

This update is also made sparse, and a suitable time window is applied. This time window starts at a small size, but its upper limit increases as $i$ grows. This way, it is ensured that strong multiples generated by interface $n$ are removed before they are (falsely) interpreted as primaries.

The sparse and time-windowed update $\langle \Delta \hat{X}_{n}^{(n)} \rangle$ is added to the primary impulse response with a scale factor $\beta$:

$$\hat{X}_{n,i+1}^{(n)} = \hat{X}_{n,i}^{(n)} + \beta \langle \Delta \hat{X}_{n}^{(n)} \rangle,$$

(3.16)
3.5. Practical Aspects

with \( \beta \) taken such, that it minimizes the objective function:

\[
\beta = -\frac{\sum_\omega \text{vec}(\Delta V_2)^H \text{vec}(V)}{\sum_\omega \text{vec}(\Delta V_2)^H \text{vec}(\Delta V_2)}, \tag{3.17}
\]

where

\[
\Delta V_2 = -c(\Delta \hat{X}_n^{(n)}) S_i + \omega(\Delta \hat{X}_n^{(n)} \hat{X}_n^{(n)} - \hat{P}_n^{(n)}).	ag{3.18}
\]

Next, \( cS_{i+1} \) is estimated as a filter obtained by least-squares matching the primary impulse responses \( \hat{X}_{n,i+1}^{(n)} + \hat{X}_{n,i+1}^{(n)} \) to \( \hat{P}_{n-1}^{(n)} + \omega \hat{X}_{n,i+1}^{(n)} \hat{P}_{n-1}^{(n)} \) in the time domain. This is done with a short matching filter (see Robinson and Treitel, 1980).

After the iterations over \( i \) for a certain \( n \), we can construct the intermediate result in a direct manner, by convolving the impulse responses with the estimated wavelet, \( P_n = c \left( \sum_{m=1}^{n} \hat{X}_m^{(n)} + \hat{X}_n^{(n)} \right) S \). Alternatively, we can produce the result conservatively by creating the multiples and subtracting them from the total data, \( P_n = P - \sum_{m=0}^{n} M_m \), where \( M_m \) is formed by Equations 2.18 and 2.20.

The intermediate results are saved, and, after incrementing \( n \), the next set of parameters is prepared by moving to the next (macro) layer (see Figure 3.1).

In Figure 3.2, an EPSI demonstration with the same three reflector model from Figure 2.8 is displayed. The intermediate (conservative) results \( P_n \) are all shown, together with synthetically generated data for comparison. Multiple events are very nicely removed. The edge artifacts introduced by the wave field extrapolation are the only visible errors in the result. The presence of these relatively small distortions allows for some minor leakage of multiple energy. However, we must note that for clean synthetic data sets like this one, edge artifacts appear much stronger in the result than in more ‘noisy’ (less sparse) field data. Moreover, because field data sets are much larger than the data sets shown here, these type of artifacts, appearing only at the edges, have a relatively small effect on the result.

3.5 Practical Aspects

Some remarks have to be made before we can implement the method described in Section 3.4 in a practical manner. A clear distinction is made between the boundaries in the subsurface, and it is assumed that the corresponding single primary events can be isolated. However, most of the time the structures in the subsurface do not adhere to the shape of a nice horizontally oriented layer, so that they cannot be separated in a clear manner.

Another fact that we should incorporate in the algorithm design is that the EPSI method is computationally intensive. We would like to compute the best results possible, but in a feasible calculation time. There is always a tradeoff between result quality and cost of resources (calculation time and computer power).

Consequently, it can be sufficient to not separate every boundary, but to define a group of boundaries, in a so called macro-layer, as visualized in Figure 3.3. When one of the two terms \( \hat{X}_l^{(l)} \) and \( \hat{X}_m^{(m)} H \) from Equation 2.19 is fairly weak, i.e., reflector \( l \) or \( m \) is fairly weak, the multiple-generating term \( \hat{X}_l^{(l)} \hat{X}_m^{(m)} H \hat{P}_n^{(n)} \) is too weak to degrade the primary data. In that case, choosing a macro layer that contains both reflector \( l \) and \( m \) does not have a dramatically negative effect on the result. Such a layer-related strategy on interbed multiple removal was also followed by Berkhout and Verschuur (2003), but in a prediction and subtraction mode.
Figure 3.2: Demonstration of a full EPSI procedure on the three reflector model from Figure 2.8. All figures are zero offset sections. The upper figures represent the real synthetically generated data. The lower figures display the EPSI results. Apart from the edge artifacts, the EPSI procedure performs very well.
3.5. Practical Aspects

Let us have another look at a full multiple removal procedure as described in Section 3.4. It is stated that, at the start, $P$ is the only known quantity, all other parameters are set to zero. It can be seen from Equation 3.15 that, in the very first iteration, $\hat{X}^{(0)}_{0,1}$ is estimated by $-PP^H$, which equals a multi-dimensional auto-correlation of the measured data. This is possible, because we use the assumption that $\hat{X}^{(0)}_{0,1} = -R_0 = I$, which, consequently, we do not have to estimate when predicting surface multiples. When the surface reflection coefficient is not $-1$, but between $-1$ and $0$, the scaling factor $\beta$ corrects for this.

However, this step (the ‘$n = 0$ step’) requires the data to actually contain surface multiples. When this is not the case, because, for example, we have measured at a situation with a very deep water bottom, or the surface multiples are too weak to really notice (which can be the case with land data), the EPSI method cannot make a good start on which to get the scheme started properly. In this case we would like to start at some other layer $n > 0$. To get the scheme going, an estimation of one of the parameters is required. Because EPSI can repair errors, this estimation does not have to be of great precision. A peak selection on $\hat{P}^{(n-1)}$ will suffice as an estimation of $\hat{X}^{(n)}_{n,1}$. However, the amplitudes of the primary impulse responses must now be controlled by minimizing the objective function from Equation 3.5 with a positive frequency-independent scale factor $\gamma$:

\[
\gamma = 1 + \frac{\sum_\omega \text{vec}(\Delta V_3)^H \text{vec}(V)}{\sum_\omega \text{vec}(\Delta V_3)^H \text{vec}(\Delta V_3)},
\]

with

\[
\Delta V_3 = \omega \hat{X}^{(n)}_n \hat{X}^{(n)}_n H \hat{P}^{(n)}_{n-1},
\]

with which we can redefine, i.e. rescale, our estimated parameters to obtain unity impulse responses:

\[
S_i := \frac{1}{\sqrt{\gamma}} S_i;
\]

\[
\hat{X}^{(n)}_{n,i} := \sqrt{\gamma} \hat{X}^{(n)}_{n,i};
\]

\[
\hat{X}^{(n)}_{n,i} := \sqrt{\gamma} \hat{X}^{(n)}_{n,i}.
\]

This step is performed at the end of every iteration, until $n$ is incremented to move to the next multiple-generating interface. During the iterations, the factor $\gamma$ converges to 1. From experience, we know that it fluctuates between 0.99 and 1.01 after the first few iterations.

Figure 3.3: Hypothetical subsurface model with complicated layer structures. Instead of trying to separate every event, two simple macro layer are designed that can be used for the interbed removal scheme. Consequently, instead of running EPSI for $n = 0..7$, a less time-consuming run takes $n = 0, a, b$. 
During the EPSI process, it can be advantageous to stop updating some parameters. The source matrix $S$, for instance, converges very fast, and often needs only 5 iterations to get to the right shape. From that point on, calculating it again in every iteration is a costly procedure without significant gain. Updating the parameter every 5 or 10 iterations gives equally good results in a shorter calculation time. Because convergence is slower for a source matrix with variable source signatures (see Appendix B), the statement above is valid for a higher iteration number.

The same holds for the parameter $\hat{X}^{(n)}$. Usually the time window of this parameter is not very large, so again convergence is quite fast. Adding the fact that this quantity was already calculated by $\hat{X}^{(n-1)}$ in a previous step (when $n > 0$), the first estimation is of high quality. Updating it in every iteration induces little gain.

In van Groenestijn and Verschuur (2009), the dipole correction factor is applied to $S$ and $P$ at the start of the EPSI procedure, saving some processing time. This way, we estimate a phase-shifted version (see Equation 2.12) of the source signature, which can easily be compensated for at the end of the estimation process.
Chapter 4

Results

4.1 Application to a Synthetic Salt Model with Shallow Water

A 2D synthetic marine data set is developed to test the EPSI method. A subsurface model, shown in Figure 4.1a, is constructed in such a way that strong surface multiples completely hide some delicate primary events from deeper down in the subsurface. Shot and receiver positions are placed every 15 meters. The shot data is generated using a 2D acoustic finite difference method.

Results of the full EPSI procedure are shown in Figures 4.1c and 4.1d. The model was divided in 3 macro-layers with boundaries at around 300m and 700m. Because no measures are taken to suppress the edge effects noted in Figure 2.5b, these artifacts allow for some leakage of multiple events.

The process of the EPSI method is illustrated in Figure 4.2 by showing some intermediate results during the iterations over $i$. One can clearly see how the (primary) impulse response is built up during these iterations.

A comparable subsurface model is used to perform the EPSI procedure without a surface removal step (proposed in Section 3.5). The synthetic data was created without reflections on the surface. At the start of the procedure, a simple estimation is made for $\hat{X}_1$ to get the scheme started properly, shown in Figure 4.3b. The result in Figure 4.3d shows that the EPSI method has some trouble to place spikes where the primaries meet the edge of the time window, indicated by the multiple black spikes around 0.7s. The primary events from our two spiky parameters overlap here. A proper design of the time windows is therefore important. This can possibly be prevented with a back propagation step to the surface just before muting, and propagating the data forward again, as was done by Berkhout and Verschuur (2005).

Some intermediate results are shown in Figure 4.4. To prevent leakage, the primary from the downward reflector must be properly estimated before the increasing time window of the parameter $\hat{X}_{1,i}$ encounters any multiple information. The upper time window is therefore fixed during the first few iterations.

The results from Figure 4.3 show less leakage than results that include the surface multiple removal step, because of the absence of artifacts in the input data (created during surface multiple removal). We may conclude that the simple first estimation is adequate enough, and that the EPSI method is robust enough to correct for possible errors in this first guess.
Chapter 4. Results

Figure 4.1: EPSI results on a synthetic salt model.

(a) Subsurface model.
(b) Stack section of synthetic data.
(c) Stack section of EPSI result after iteration \( n = 0 \), i.e., after removing all surface multiples.
(d) Stack section of EPSI result after iteration \( n = 2 \).

In (a), the primary events from the three weak reflectors around 800 m deep are covered by the strong surface multiples from the water bottom and the upper subsurface boundaries. The large tails sticking out from the sides in (c) and (d) are edge effects, which have not been accounted for during the process. The presence of these artifacts cause some leakage of multiple energy. Because the data used shown in this chapter consist of completely filled matrices, the CMP gathers at low and high midpoint position have a low trace count. Consequently, the noise-cancelling stacking operation has limited effect at the edges.

The large tails sticking out from the sides in (c) and (d) are edge effects, which have not been accounted for during the process. The presence of these artifacts cause some leakage of multiple energy. Because the data sets shown in this chapter consist of completely filled matrices, the CMP gathers at low and high midpoint position have a low trace count. Consequently, the noise-cancelling stacking operation has limited effect at the edges.
4.1. Application to a Synthetic Salt Model with Shallow Water

Figure 4.2: Illustration of the progress of the EPSI procedure during surface multiple removal. (a) Shot record from the original data. (b) The first estimation of the primary impulse response is constructed by an auto correlation of the data; \( PP^H \). (c) After imposing the sparseness constraint on the data in (b), we obtain \( X_{0,1} \). (d)-(e) Intermediate results \( X_{0,i} S_i \) for \( i = 1, 5 \) and 10, respectively. (After van Groenestijn and Verschuur, 2009)
Figure 4.3: EPSI results on a synthetic salt model, comparable to the model in Figure 4.1. (a) Shot gather without surface multiples. (b) First estimation of the impulse response of the downward reflector. This estimation was done by a peak selection of the hyperbolic Radon transform of the CMP gathers of the original data. (c) EPSI result after one interbed removal step ($n = 1$). (d) Corresponding primary impulse response.
4.1. Application to a Synthetic Salt Model with Shallow Water

Figure 4.4: Illustration of the progress of the EPSI procedure directly applied to surface multiple free data. (a)-(c) Shot records of the estimated primary of the downward reflector for iterations 1, 10 and 30. (d)-(f) Shot records of the estimated primaries from the boundaries below the downward reflector for the corresponding iterations. In the first iteration, $\hat{X}_{1,1}^{(1)}$ is estimated by a peak selection of the hyperbolic Radon transform of the CMP gathers. A converged version of $\hat{X}_{1,1}^{(1)}$ is needed to properly predict the interbed multiples. Consequently, the upper time window on the $\hat{X}_{1,1}^{(1)}$ parameter is not increased during the first 10 iterations.
4.2 Field Data Application: Vøring Basin

To test the multiple removal capacities of the EPSI method in a real situation, a field data set from the Vøring basin in Norway was provided by Statoil ASA. The basin has a relatively deep water bottom, a situation to which the surface removal capacities of EPSI had not been tested before.

Because of the deep water situation, the early reflections appear pretty horizontal in the shot records. It is therefore safe to assume that a Radon interpolation step to construct the missing near offset data is accurate. This has been done before applying multiple removal.

Figure 4.5 shows stack sections of the original data, and time-windowed stack sections of comparable SRME and EPSI (only surface multiples removed) results. The SRME method was applied to the whole data set, with CMP numbers ranging from 0 to 900 and time from 0 to 8 seconds. Because of the limitations of the current EPSI implementation, the latter could only be applied to a small subset of the total data, with only 101 source and receiver positions (compared to 185 positions for the SRME implementation). All images in Figures 4.5 and 4.6 have been windowed to fit this subset before stacking.

Because EPSI is applied to a subset (both in time and space) smaller than SRME, one expects the former to show more leakage than a full (moving spread) SRME application. The restriction in the slow dimension (position) allows boundary artifacts to show up, while the time window limits the accuracy of the correlation step converting the multiples to primaries (see Section 3.4). Indeed, from Figure 4.5c, it can be concluded that the EPSI result has trouble removing multiples at the edges. Still, EPSI seems to remove the surface multiples better than the SRME method after we apply a stacking operation on the CMP sections, as shown in Figure 4.5b. Apparently, the minimum-energy constraint that SRME uses does not provide the right result.

To compare the interbed removal qualities of EPSI, comparable procedures (EPSI and SRME + IME) are performed on the resulting data sets from above. A macro boundary is placed at around 2.3s to remove all interbed multiples that passed that boundary directly before and after their downward reflection. The results of the above procedure are displayed in Figure 4.6. The IME results of this data were reported in Verschuur and Berkhout (2005).

The great similarity between the results in Figures 4.6c and 4.6f is on the one hand a validation of the introduced interbed multiple removal method applied to field data sets. On the other hand, without noticeable difference, it can be argued that the slow and resource consuming EPSI method has little added value over the relatively fast method of IME. The small difference between the two results can be explained by the fact that the predicted interbed multiples lie in regions with weak primaries, so the L2 norm that proved incorrect before is a safe assumption in this case. When predicting interbed multiples that overlap with strong primaries (as conducted below), the IME result will be worse.

A full EPSI procedure is conducted to remove all multiples generated by the upper layers that appear in the measurement data between 1.6 and 2.7 seconds. To perform a multiple removal step as thorough as possible, six macro-layers are distinguished. The boundaries are placed between the various primary events in the specified region. Results are shown in Figures 4.7 and 4.8.
Figure 4.5: Comparison of surface multiple removal results of EPSI and SRME on a deep water marine data set, provided by Statoil ASA. (a) Stack section of the (subset of the) measured data. (b) Stack section of the conservative result from EPSI. (c) Stack section after subtraction of pre-stack predicted multiples by SRME. (d) Shot record of the measured data. (e) Shot record of the conservative EPSI result. (f) Shot record of the SRME result. From (e) and (f), it follows that SRME nor EPSI can fully remove all surface multiples. Though, when stacked, the EPSI result shows less leakage (for instance, compare the indicated positions in (b) and (c)). The EPSI result in (e) shows little leakage in the low offset region (for instance, compare the indicated positions in (e) and (f)). For a better comparison, figures (b), (c), (e) and (f) are time-windowed to zoom in at the moment of appearance of the first order surface multiples.
Figure 4.6: Comparison of interbed multiple removal results of EPSI and SRME + IME on the same field data set as shown in Figure 4.5. For the interbed removal step, a macro-boundary has been placed at around 2.3s. The curvature of the muting windows match the curvature of the primary events. All figures are stack sections. Figures (c) and (f) are amplified with a factor 10.
4.2. Field Data Application: Vøring Basin

Figure 4.7: Demonstration of a full EPSI procedure on the Vøring basin data set. All figures are stack sections and have the same clipping values for a fast comparison. (a) Original measurement data (same as Figure 4.5a). (b) EPSI result after surface removal step; $P_0$. (c) EPSI result after 6 interbed multiple removal steps; $P_6$. This is the ‘multiple free’ data. (d) Removed surface multiples by EPSI; $M_0$. (e) Removed interbed multiples after interbed multiple removal 6 steps; $\sum_{n=1}^{6} M_n$. Some positions are indicated in (b), (c) and (e) where interbed multiple events are visibly removed.
Figure 4.8: Time-windowed versions of the results shown in Figures 4.7b, c and e, with higher clipping values. The removed interbed multiples are clearly visible. (a) EPSI result after surface removal step; $P_0$. (b) EPSI result after 6 interbed multiple removal steps; $P_6$. This is the ‘multiple free’ data. (c) Removed interbed multiples after interbed multiple removal 6 steps; $\sum_{n=1}^{6} M_n$. The same positions are indicated as in Figure 4.7.
Chapter 5

Conclusions

The EPSI method was previously introduced as a surface multiple removal method. It is a large scale inversion process, in which primaries are estimated, which, together with their corresponding surface multiples, explain the total data. The primaries are parameterized as a collection of spikes and a source signature, the update on the spiky parameter is made sparse. The use of a new physical model provided better results than contemporary surface multiple removal methods like SRME.

We have successfully extended the EPSI method to provide interbed multiple removal. The original EPSI method can be seen as a special case of the new, more generalized EPSI method, being the step for \( n = 0 \). The new method has been applied to a synthetic data set and a field data set, and produced satisfactory results.

The EPSI method can be applied such that it does not use surface multiples. This is valuable for land data with very weak surface multiples. In this case the method needs a first estimation of the primary impulse response. The estimation does not have to be very precise, making it a robust method.

The EPSI method is a good alternative next to SRME and IME. The basic parameterization differs, making EPSI a better model in many cases, producing better results. The adaptive subtraction, which uses a minimum energy (L2) norm, in SRME and IME is omitted in EPSI. Thereby, the EPSI method improves the result in particular when this norm is not met in the situation addressed.

There are, however, some downsides on the EPSI method. First of all, it takes more time to process. Instead of implementing all possible steps, one can decide to speed up the calculations by assuming no further improvement of certain parameters.

A problem that might prove harder to solve is the use of the various muting windows that isolate or suppress certain events. Besides the fact that some events can be very hard to isolate because of their complex shapes, this means that there must be some a priori knowledge or at least a proper steering of the procedure during the iterations.

To conclude, we can say that the EPSI method improves results on multiple removal. However, its applications should be chosen with care. Only in situations where the SRME or IME assumptions might prove incorrect can the EPSI method really gain quality over the ‘old’ methods.
Chapter 6

Recommendations

The proposed generalized EPSI method should not be interpreted as a ‘plug and play’ procedure to remove all multiples from any data set. The development of the method is a small contribution to the whole field of seismic multiple removal research, and it should be treated that way. This means that, besides the further development of the theory, design and implementation of the generalized EPSI method, the findings from this thesis could be used to improve similar multiple removal techniques.

In the short term, the next step that should be taken in this research is to implement EPSI in a fast, efficient way, that can handle large data sets. The current implementation was written in MATLAB, which significantly limits memory usage and processing speed. A C or C++ implementation would resolve these problems. Only when we are able to perform a moving spread procedure on a full field data set can we really compare EPSI with the established methods. Because of the multiple correlation steps included in the procedure, the quality of the result gets higher when more data is available. Afterwards, a study should be made to investigate in the benefits (and possible limitations) of EPSI over SRME/IME. Knowing when to use which method can save a lot of time in a practical situation.

A possible time saver on the current implementation of EPSI has to do with the band limitation imposed on the impulse responses. We have restricted ourselves to addressing spikes on a time grid that matches the sampling rate of our signal. However, in the hypothetical situation that we know that the source signature has limited frequency, we would be able to perform an interpolation step on the update of our impulse response just before applying our sparseness constraint. This increases the temporal resolution of the parameter, and can greatly improve the convergence of the objective function.

In Section 2.6 it was mentioned that the construction of interbed multiples used in the EPSI method introduces an error by the lack of compensation for transmission effects up to and including the multiple generating boundary. Though in our efforts to investigate the performance of EPSI nothing indicated the presence of such errors, a small investigation could be made to measure the impact of this inconsistency. If needed, countermeasures could be designed to neutralize the (most likely negative) effects.

In van Groenestijn and Verschuur (2009) the EPSI method can replace the near offset reconstruction step. This part of the theory was not adopted in this thesis, because in many cases the interbed multiples are too weak to reconstruct the primaries. In a practical situation, the surface multiples would be used to reconstruct near offsets in the $n = 0$ step, but not in interbed multiple removal steps. However, the theory can be extended to reconstruct missing primaries from interbed multiples. This can be used in situations with very strong interbed multiple generators, and weak surface multiples.
Another obvious step is to extend the generalized EPSI method to three dimensions. It was recently shown by Savels et al. (2010) that in the presence of cross-line dip, the 2D SRME method generates better results than the 2D EPSI method. Therefore, a 3D approach is necessary for an effective EPSI application.

As mentioned before, a weak point of the EPSI method is the usage of muting windows. These must be constructed beforehand, and influence the result. A possible improvement can be made by including a back propagation step from the chosen depth level before muting, and propagating the data forward again, as was done by Berkhout and Verschuur (2005). In theory, this improves the separation of events, especially in situations where primary events cross each other at large offsets. However, the improvement would be limited by the argument that the majority of multiples are predicted by the lower offset regions.

A better solution is to totally exclude the muting windows by adapting the method. An example is the recent work on the EPSI method by Lin and Herrmann (2010), where the inversion is stabilized using convexification, omitting the time window that contains multiple-free primaries and some other inversion parameters from the procedure. Extending this research to the generalized EPSI method might prove valuable.

The above improvement might help us to optimize the generalized EPSI procedure. The current implementation removes multiples from one subsurface (macro) layer at a time. The updates on the parameters are formed from contributions from the unexplained primary information and from the unexplained multiple (generated by the current macro layer only) information in the residual (see Section 3.4). A faster procedure would estimate primaries from information of all multiple information, and also predict all multiples in one go with an extended inversion step, similar to work by Weglein et al. (1997). Further research can determine whether this is possible for EPSI in a stable way.

Van Groenestijn and Verschuur (2010b) recently showed that EPSI could be used to deblend time-overlapping seismic measurements. These type of measurements allow for shorter acquisition time, or improve illumination in the same acquisition time. The concept of blended shot data shares the notion with EPSI that the higher the illumination, the better the information, regardless the direction from where the information came from. Research should be conducted to extend the generalized EPSI method to blended shot data.

The concepts of the EPSI method can be used in other ways of parameterization. At the moment, we use an impulse response with a sparseness constraint and a wavelet in a physical model to explain our measured data. But the underlying problem we are tackling is not about extracting an impulse response, instead, we would like to gain knowledge about the subsurface. Taking one step (or several) further, we might end up with a parameterization of the actual subsurface. Full waveform inversion methods could translate the measured data directly to medium parameters, taking all primaries and multiples into account in one step. This concept has well been researched in the past years. An overview of the current progress on this challenging procedure was described by Virieux and Operto (2009). Though realistic applications are becoming possible today, it is stated that much research is needed before we are able to improve this technique up to the required standards.
Chapter 7

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Appendix A

Matrix Notation

A 2D seismic full monospaced measurement results in a matrix of traces - forming a cube-like structure - with the source position and receiver position sorted along the horizontal and vertical axis, respectively (see Figure A.1). For every source position in the measurement field the response of the subsurface has been measured on all receiver positions. The measured data $P(x_r, x_s, \omega)$ (which is the temporal Fourier transformed data $p(x_r, x_s, t)$) is organized in the matrix $P(\omega)$. Note that throughout this thesis we omit the $\omega$ dependence and denote the data with $P$, assuming that all described steps are repeated for all frequency components. Each column of the matrix $P$ contains a shot record and each row contains a common receiver gather. We can obtain common offset sections from parallels to the main matrix diagonal (being the zero-offset section), and common midpoint gathers on the gathers that are oriented perpendicular to the main diagonal.

![Figure A.1: Schematic display of the data organization. When a slice is taken from a full measurement cube, a matrix is obtained. From this matrix we can distinguish shot records, common receiver gathers, common offset sections and common midpoint gathers.](image)

Surface multiples are predicted by adding one round-trip to all reflectors and up again to the measured data. As described in Section 2.4, a double convolution process can produce the desired results. The process can be described in a single step, by acknowledging the fact that the Green’s functions that are used can be replaced by the dipole impulse response $X_0$. A multi-record convolution of the impulse response with the total data delivers the surface multiples. In the integral form of the
Appendix A. Matrix Notation

Figure A.2: Surface multiples are predicted with a matrix multiplication. One output point of the multiple matrix $M_0$ can be obtained by the vector multiplication of a row of $X_0$ with a column of $P$.

2D Rayleigh II integral (Equation 2.11) this looks like:

$$M_0(x_r, x_s, \omega) = -\int_{x_k} X_0(x_r, x_k, \omega) P(x_k, x_s, \omega) dx_k,$$

where $x_r$ is the receiver position, $x_s$ is the source position and $x_k$ is the lateral position over which the summation takes place. We have assumed the reflection operator of the surface to be -1 for all frequencies.

By the fact that the data is discretized in every direction, in practice, the integral formulation of surface multiple prediction, as given by Equation A.1a, gets replaced by a summation:

$$M_0(x_r, x_s, \omega) = -\sum_{x_k} X_0(x_r, x_k, \omega) P(x_k, x_s, \omega).$$

When constructing a single trace in $M_0$, a common receiver gather from $X_0$ is taken, and a shot record from $P$. By multiplying the source-receiver pairs in the frequency domain, a convolution of the corresponding time-signals is performed. All possible ray path combinations are considered, as shown in Figure A.2. The summation eventually ensures that only the stationary contribution remains, which is demonstrated in Figure 2.4.

When the above process is repeated for all source and receiver positions, but only for one frequency, thus filling every element of the matrix $M_0$, this can be written as a matrix multiplication:

$$M_0 = -X_0P,$$

which is illustrated in Figure A.2. Consequently, the matrix multiplication has to be performed for each frequency. Optionally, the results need be inverse Fourier transformed to obtain the output in the time domain.
Appendix B

Varying the Source Signature

B.1 Introduction

The EPSI method has been presented by van Groenestijn and Verschuur (2008) with an assumption that the source signature is constant for all shots. Also, it is assumed that the sources all radiate the same in all directions. In Section 3.2, however, it was mentioned that by using a diagonal matrix or a band matrix for the source wavelet $S$, source-to-source wavelet variations and source directivity could be included.

As a part of this research, a small investigation on the possibility of including the source variations was performed. By replacing every scalar $S$ with a matrix $S$, and muting out the parts that need to be kept zero when updating $S$, the extension is realized in a fairly simple manner (a same research was conducted by van Groenestijn and Verschuur (2010a), but without field data results).

B.2 Testing on a Synthetic Data Set

A synthetic data set has been constructed to test the EPSI procedure with a varying shot signature. Each group of 25 shots has a different signature. The three reflector model was used with four different source signatures. The result can be found in Figure B.1.

Apart from the fact that, during the iterations, the result seems to converge a bit slower than in the constant source situation, the final result fits the expected data very nicely.

B.3 Application to a Field Data Set

The synthetic results look promising. Let us now see whether the varying source model can make a difference on field data. A surface multiple removal step was performed on the Vøring basin data set. Results are shown in Figure B.2.

As we can see there is little difference between the fixed source and variable source results. The difference between Figures B.2a and B.2d is amplified in Figure B.3. We can see that a substantial part of the difference consists of edge artifacts. The extra degrees of freedom we gave the EPSI model by allowing for varying source signatures, are trying to compensate the error introduced by the edge artifacts. The. This reduces the objective function in Equation 3.5, but is does not lead to a better
multiple prediction.

B.4 Conclusions

We can conclude that we must be careful on the decision to include a varying source signature. We should use this version of the EPSI model only in cases where the source signature really has a significant variation. In other cases, the varying source signature assumption can degrade the result.

What we did not note yet, are the practical consequences of this implementation of the EPSI model. While the EPSI model is computationally intensive, changing the source signature from \( SI \) to a diagonal matrix \( S \) has not that much impact on the memory load or processing time. When we would expand the diagonal matrix to a band matrix, the CPU load will increase significantly.

The results in this chapter discourage us to use a varying source signature throughout the other parts of this thesis. However, we still opt to include the full matrix notation in our mathematical descriptions for completeness.
Figure B.1: Result of the EPSI procedure on the three-reflector model. (a) Zero offset section of the synthetic data set, with 4 different wavelets. (b) Zero offset section of the estimated primaries after the full EPSI procedure. (c) Zero offset section of the estimated primaries after the full EPSI procedure on the same subsurface model, but with a constant source wavelet (see also Figure 2.8). (d) Zero offset section of the estimated primary impulse response after the full EPSI procedure. (e) Zero offset section of the estimated primary impulse response after the full EPSI method, performed on the constant source data. The result on the varying source method looks promising, though there is some more leakage than with the constant source data. This can be caused by the fact that the estimated primary impulse response in (d) is less ‘clean’ than the one seen in (e).
Appendix B. Varying the Source Signature

Figure B.2: Comparison of the surface multiple removal step of EPSI using either a constant source or a varying source signature. (a) Stack section on $P_0$ from EPSI assuming a constant source wavelet. (b) Stack section on corresponding removed surface multiples. (c) The estimated source wavelet. (d) Stack section on $P_0$ from EPSI assuming a varying source wavelet. (e) Stack section on corresponding removed surface multiples. (f) The estimated source wavelets in the main diagonal of $S$. Differences between the two results are very small. Apparently, at least in this case, the constant source signature is a safe assumption.
Figure B.3: Difference (amplified by a factor 10) between the stack sections of the surface removal step with constant and varying source signature, see Figure B.2. Edge artifacts introduced by wave field extrapolation show up relatively strong here.
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